AIAA JOURNAL
Vol. 44, No. 12, December 2006

Extensive Experiments on Genetic Algorithms for the Optimization of Piezoelectric Actuator Locations

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DOI: 10.2514/1.19181

For shape control of smart structures, we have developed a series of micro-genetic-algorithms for optimal placement of a large number of piezoelectric actuators. Here, we investigate the effect on the solution quality of 1) different random seed generators; 2) restart criteria in micro-genetic-algorithms, specifically the two parameters, the number of generations used for the inner loop and the level of diversity in the population; and 3) different numbers of actuators. We also report a comparison of our genetic algorithms with two heuristic integer programming algorithms: worst-out-best-in and exhaustive single point substitution proposed by Haftka and Adelman ("Selection of Actuator Locations for Static Shape Control of Large Space Structures by Heuristic Integer Programming," *Computers and Structures*, Vol. 20, 1985, pp. 572–582). Using current genetic algorithms, we not only get better layouts of actuators than reported in our previous publications, but we also find that the most distinct nature of genetic algorithms is randomness and robustness. The best parameter setting of genetic algorithms is dependent on both the number of evaluations used for termination and the seed generator used. Moreover, the best parameter setting of genetic algorithms varies as the number of actuators changes. To get the highest quality of solutions, multiple runs using different random seed generators are necessary. The time of investigation can be significantly reduced using a coarse grain parallel computing. Comparison of our genetic algorithms with the worst-out-best-in and exhaustive single point substitution shows that for a group of runs our genetic algorithms usually converge faster in the first few thousand evaluations and are more likely to find better solutions than the worst-out-best-in and exhaustive single point substitution algorithms.

Nomenclature

A	=	coefficient matrix of system equations
$\boldsymbol{b}_k$	=	right-hand vector of system equations
d_{ij}	=	piezoelectric coefficients, m/V
E	=	rms error of primary mirror surface distortions
L	=	location of piezoelectric actuators
m	=	number of nodes of the finite element model
n	=	number of piezoelectric actuators
T	=	thermal loads
u_i	=	correction to the transverse displacement
V_j	=	voltage applied on the piezoelectric actuators
w_i	=	thermal deformation (transverse displacement)
α_{ij}	=	influence coefficients

I. Introduction

DURING the last two decades, research and development of smart structures has received significant attention from university, government, and industry researchers [1–9]. An example is the design of the next generation of astronomical telescopes [10]. One of the most stringent requirements for these telescopes is the maintenance of a very high surface accuracy of the primary mirror during their operation. A promising method is to use a certain number of piezoelectric actuators bonded onto the rear surface of the primary mirror to correct its distortions without imposing a significant weight penalty. The critical issue is how to arrange these actuators to maximize their benefit. If the number of actuators is small, we could easily solve the problem by using the exhaustively enumerative search method, that is, by checking every possible solution in the search space. But this is not possible for large problems. Usually, a large number of actuators are needed for the structure to operate properly, and so there are an extremely large number of possible schedules for placing these actuators. In our previous studies [11,12], we developed a methodology of combining the finite element method and genetic algorithms to solve this challenging problem.

We have successfully developed a series of genetic algorithms (GAs), including a regular GA and micro-GAs, termed versions 1, 2, and 3. With limited runs, we showed some very promising results produced by the latest version of our micro-GA codes, termed version 3 [12]. As pointed out to us by a reviewer of our previous work that was also published in this journal, because genetic algorithms are random in nature, the results may be a matter of chance and thus must be verified by performing a large number of runs. In this paper, we study the GAs by performing such an extensive numerical verification. With this study, using a large number of runs, we aim to get a better understanding of genetic algorithms, which have emerged as a leading global search method for various complex problems. Our research shows that the most distinct nature of genetic algorithms is randomness and robustness.

Throughout the GA literature, we have not found the paper that gives the reader a clear concept, what the randomness and robustness of GAs really means. We want to provide such a concept here. With runs of 15,000 evaluations, the GA version 3 found a nearly optimal solution from the order of 10^{21}–10^{56} possible solutions in all of our test cases. However, to get the highest quality of solution, a number of runs using different seed generators are necessary. We also report the limited comparison of our GAs with two heuristic integer programming algorithms: worst-out-best-in (WOBI) and exhaustive

Presented as Paper 1899 at the AIAA/ASME/ASCE/AHS/AHC 46th Structures, Structural Dynamics and Materials Conference & 1st AIAA Multidisciplinary Design Optimization Specialist Conference, Austin, TX, 18–21 April 2005; received 28 July 2005; revision received 28 August 2006; accepted for publication 6 September 2006.

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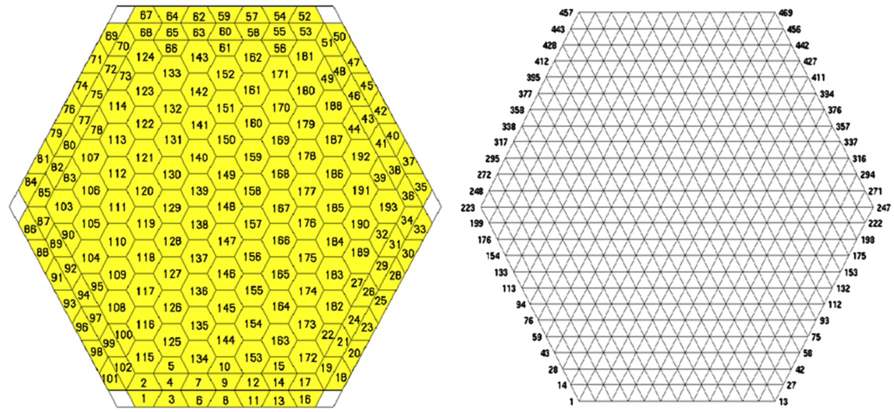


a) Piezoelectric actuator candidate locations b) Finite element mesh

Fig. 1 Thin hexagonal spherical primary mirror segment.

Table 1 The main differences among the regular GAs, micro-GA version 1, micro-GA version 2, and micro-GA version 3

	Regular GAs	Micro_GA version 1	Micro-GA version 2	Micro-GA version 3
Restart function	No	Yes	Yes	Yes
Crossover	Yes	Yes	Yes	Yes
Mutation	Yes	No	No	Yes
Individual replaced by the best solution	——	Randomly selected individual	The worst individual	The worst individual
Size of population	Large, fixed	Small, fixed	Small, variable	Small, variable
Hill climbing	——	No	No	Yes

single point substitution (ESPS) proposed by Haftka and Adelman in the early 1980s [13].

II. Genetic Algorithms

Genetic algorithms, inspired by biology and population genetics, are increasingly being accepted as an efficient optimization method to solve a wide variety of problems as shown by the ever-growing number of publications dedicated to this subject [14–26]. Even though there are various GAs proposed, they can be simply classified into two categories: regular GAs and micro-GAs. In their structure, the micro-GAs are the same as the regular GAs except that they include an automatic restart function when the population converges to a solution according to some convergence criterion. The

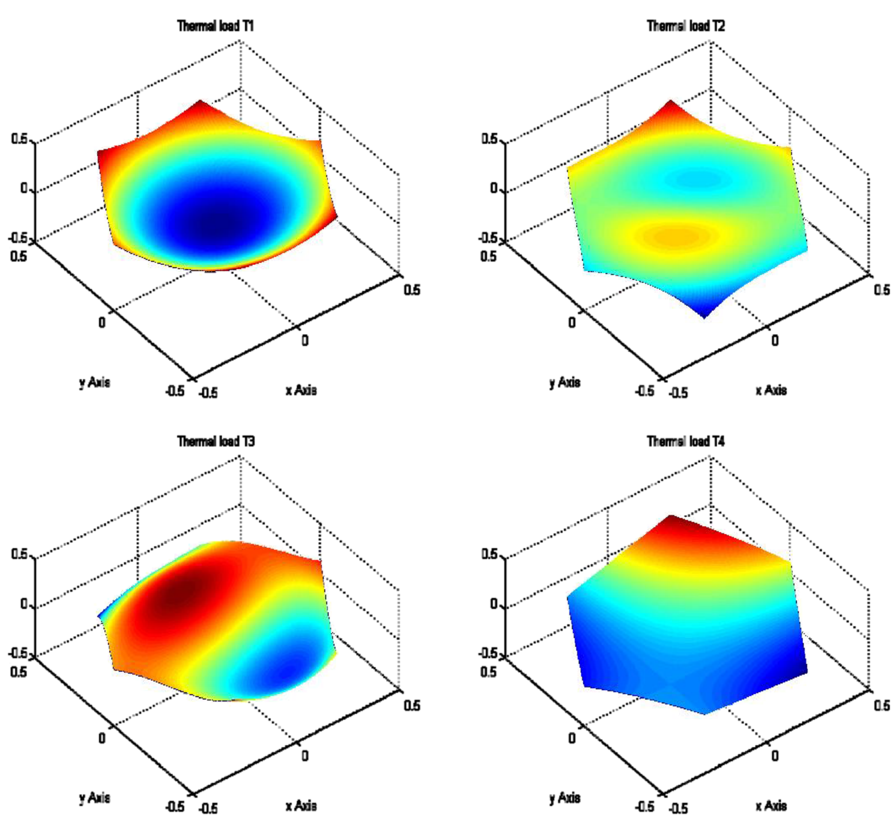


Fig. 2 Thermal loads $T1$, $T2$, $T3$, and $T4$.

Table 2 Properties and geometry of the mirror and piezoelectric actuators

	Mirror (beryllium)	Piezoelectric strips
Young's modulus, GPa	293	63
Poisson's ratio	0.1	0.3
Coefficient of thermal expansion, /°C	11.5×10^{-6}	0.9×10^{-6}
$d31$, $d32$, m/V	——	254×10^{-12}
Radius, m	10	——
Side of the hexagon, m	0.5	0.04166
Thickness, m	0.012	0.25×10^{-3}

Table 3 Temperature distributions at the lower surface of the mirror [10]

	Temperature distribution
$T1$	$C[2(x^2 + y^2) - 1]$
$T2$	$C[(x + y)(3x^2 + 3y^2 - 2)]$
$T3$	$C(\sum_{i=1}^{i=9} K_i Z_i)$[a]
$T4$	$C[x + y + 2xy]$

[a] K_i, from Table 2 of [27], p. 226 (back surface) and Z_i, terms of Zernike series, [27], p. 216.

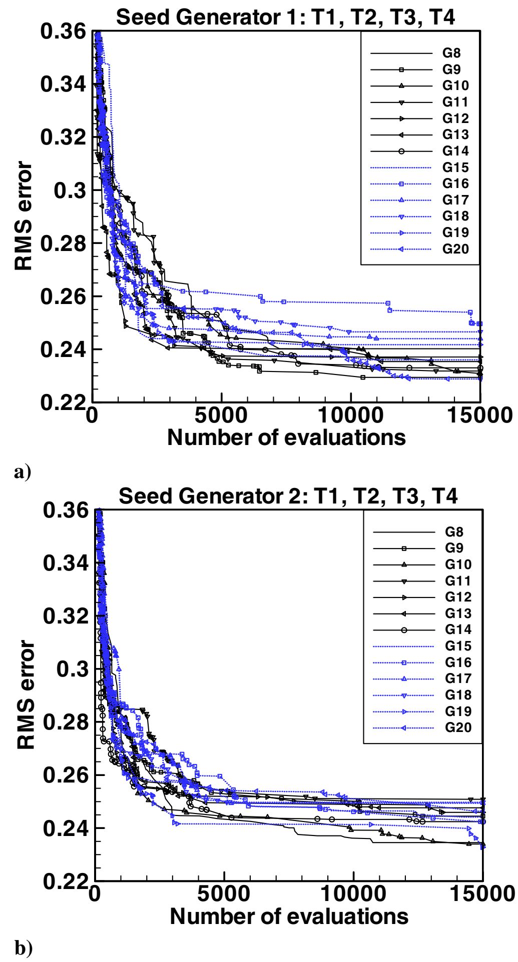


Fig. 3 Performance of the GAs: 30 actuators.

significant difference between regular GAs and micro-GAs is that the micro-GAs usually use a much smaller population size to achieve a faster convergence rate than the regular GAs that use a large population size. In the GA literature, regular GAs have received more thorough study with respect to the choice of various parameters than micro-GAs. However, our experience shows that micro-GAs are much more efficient than regular GAs. Moreover, micro-GAs also benefit from the mutation operation. In this paper, we will study the GAs by extensive numerical experiments and further demonstrate that our GA version 3 is an efficient, reliable, and robust optimization tool for the challenging problem of choosing the optimal location for a large number of actuators or sensors in the design of the next generation of smart structures. The results by running our latest micro-GA version 3 on the high-performance Sun machine with 17 processors are reported in this paper. The flowchart for the micro-GAs is available in [26], and a more detail description of our micro-GA version 3 is available in [12]. The main differences among the regular GAs, micro-GA version 1, micro-GA version 2, and micro-GA version 3 are shown in Table 1. The two heuristic integer programming algorithms WOBI and ESPS are available in [13].

III. Problem Definition

The optimization problem investigated in this study is as follows:

Given n piezoelectric actuators, find an optimal placement of these actuators from a possible 193 candidate locations to get the best correction to the surface thermal distortions under a combination of four different types of thermal loads in a thin hexagonal spherical mirror segment to be used in the next generation of astronomical telescopes (Fig. 1a). That is, the problem is to find only one set of locations and the corresponding individual set of voltages that provide us the best correction for all four different thermal distortions. Note that only the locations of actuators are the same for all four types of loads, not their voltages. This is a challenging multicriterion optimization problem. The total number of different candidate sets can be computed by the following formula:

Table 4 The rms errors corresponding to optimal placement (30 actuators)

Generations of inner loop	rms for initial evaluation		rms for 5000 evaluations		rms for 10,000 evaluations		rms for 15,000 evaluations	
	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2
8			0.25069	0.24362	0.24022	**0.23622**	0.24022	0.23455
9			**0.23541**	0.25342	**0.23130**	0.25026	0.22943	0.24451
10			0.24551	0.24459	0.24102	0.23923	0.23093	0.23399
11			0.23865	0.25410	0.23581	0.25116	0.23186	0.25083
12			0.23831	0.25208	0.23709	0.25020	0.23709	0.24594
13			0.24040	0.24958	0.23924	0.24892	0.23538	0.24764
14	0.87205	0.61079	0.25305	0.24685	0.23455	0.24328	0.23304	0.24243
15			0.24089	0.25215	0.23744	0.24655	0.23604	0.24417
16			0.26163	0.25954	0.25789	0.24619	0.24962	0.24241
17			0.24749	0.25167	0.24473	0.24938	0.24399	0.24938
18			0.25494	0.25040	0.24657	0.24971	0.24657	0.24971
19			0.24287	**0.24162**	0.24175	0.24162	0.24175	**0.23277**
20			0.25131	0.25439	0.23615	0.25130	**0.22882**	0.24636

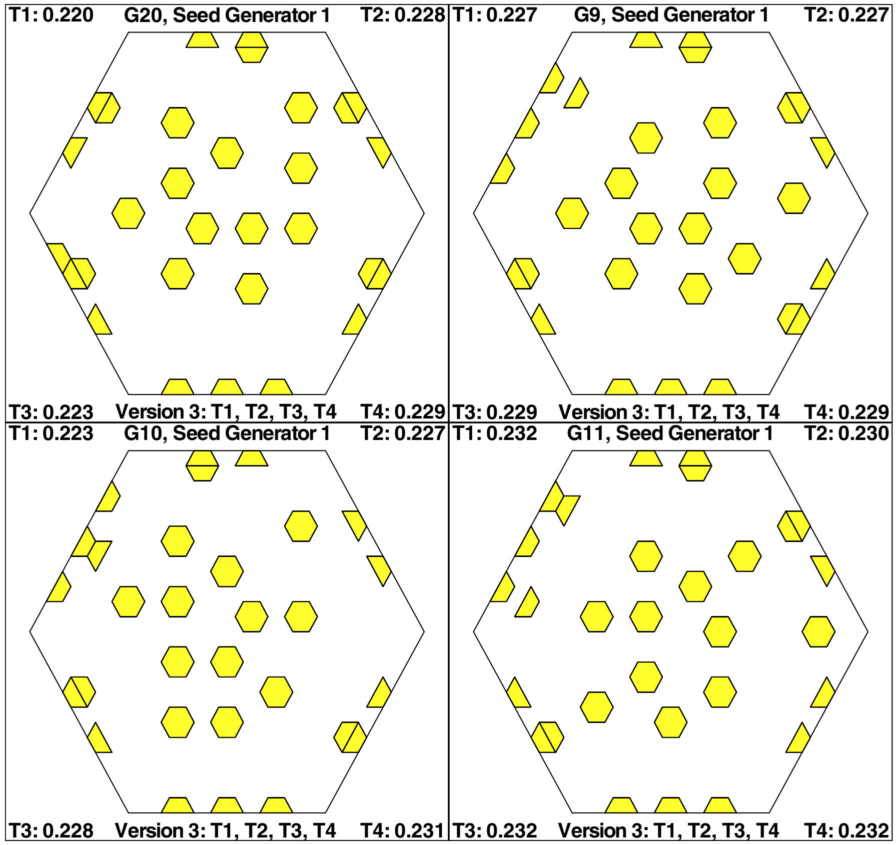


Fig. 4 RMS error and optimal location of 30 piezoelectric actuators.

$$ {}^{193}C_n = \binom{193}{n} = \frac{193!}{n!(193-n)!} \tag{1}$$

To test the stability and reliability and to better understand the performance of our GA version 3, a set of a given number of actuators will be tested. Specifically, we tested the following:

Case 1:

$$ n = 30, \qquad {}^{193}C_{30} = 1.28866 \times 10^{35} $$

Case 2:

$$ n = 121, \qquad {}^{193}C_{121} = 1.38231 \times 10^{54} $$

Case 3:

$$ n = 15, \qquad {}^{193}C_{15} = 8.4 \times 10^{21} $$

Case 4:

$$ n = 60, \qquad {}^{193}C_{60} = 5.53626 \times 10^{50} $$

Case 5:

$$ n = 90, \qquad {}^{193}C_{90} = 4.65676 \times 10^{56} $$

It is clearly seen that it is impossible to use the exhaustive enumeration method to find the solution from such a huge number of candidate sets even with the most advanced computer in the world.

The geometry and material properties of the mirror and piezoelectric actuators are given in Table 2. The temperature distribution at the lower surface of the mirror is assumed to be in the form of a linear combination of the first few terms of the Zernike series expressed in terms of Cartesian coordinates x and y with the origin at the center of the mirror. The Cartesian coordinates used to

Table 5 The rms errors corresponding to optimal placement (121 actuators)

Generations of inner loop	rms for initial evaluation		rms for 5000 evaluations		rms for 10,000 evaluations		rms for 15,000 evaluations	
	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2
8			0.07880	0.08017	0.07825	0.07891	0.07802	0.07864
9			0.07930	0.07933	0.07845	0.07874	0.07817	0.07831
10			0.07982	0.07894	0.07942	0.07822	0.07931	0.07810
11			0.07927	0.07951	0.07862	0.07843	0.07824	0.07813
12			0.07866	0.07872	0.07828	0.07812	0.07805	0.07797
13			0.07881	0.07850	**0.07802**	0.07817	**0.07788**	0.07809
14	0.20413	0.18827	0.07897	0.07918	0.07854	0.07868	0.07851	0.07853
15			0.07970	0.07902	0.07860	0.07826	0.07831	0.07815
16			0.07903	0.07879	0.07871	0.07855	0.07857	0.07822
17			0.07893	0.07860	0.07840	0.07822	0.07810	0.07809
18			0.07898	0.07892	0.07858	0.07870	0.07832	0.07857
19			0.07861	0.07839	0.07852	0.07831	0.07790	0.07813
20			0.07908	**0.07833**	0.07888	**0.07799**	0.07877	**0.07795**

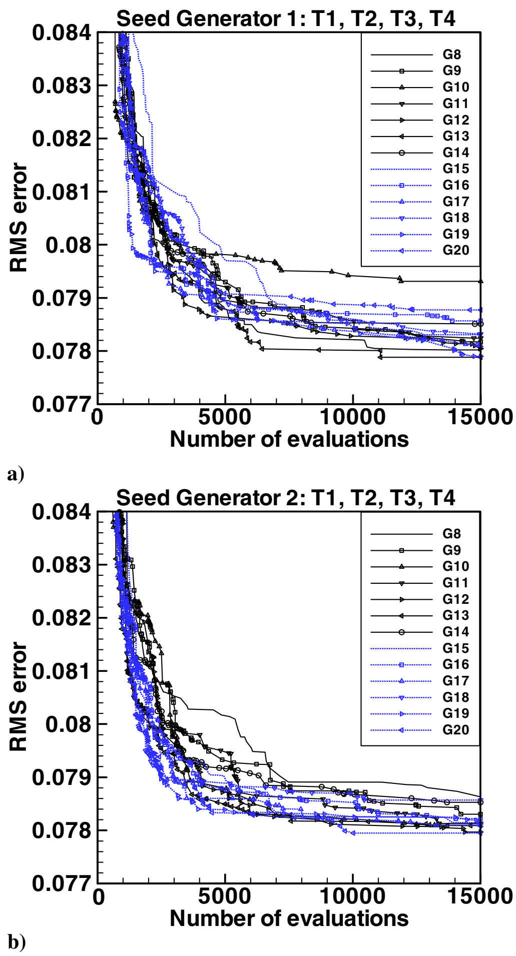


Fig. 5 Performance of the GAs: 121 actuators.

express the temperature distributions are normalized such that they are in the range $[-1, 1]$. The temperature distributions that are considered in this study are given in Table 3 (they are the same as those in Table 5 of [10]), where the constant C is used to scale the temperature distributions such that the upper light-reflecting surface is at a lower temperature than the lower surface, with a constant temperature difference ΔT_z°C, and the maximum temperature difference between any two grid points across the lower surface of the mirror is ΔT_{xy}°C. In this study $\Delta T_z = 0.2$°C and $\Delta T_{xy} = 0.5$°C. The graphic representation of these thermal loads is shown in Fig. 2.

IV. Finite Element Modeling

A laminated triangular shell element [28] is used to model the mirror. The element is a combination of the discrete Kirchhoff theory plate bending element and a membrane element derived from the linear strain triangular element with a total of 18 degrees of freedom (three translations and three rotations per node). The piezoelectric strips are assumed to be perfectly bonded to the lower surface of the mirror and are modeled as a separate layer. The finite element model consists of 864 flat shell elements and 469 grid points (Fig. 1b). The mirror segment is assumed to be simple-supported at the six vertices 1, 13, 223, 247, 457, and 469.

V. Control Algorithms

The surface thermal distortions or the transverse displacements w of the mirror segment are corrected by applying the voltage across the thickness of the strip, which induces a distributed strain in the strip and hence in the mirror. In this study, the thermal deformation due to a given type of thermal loads is computed by the finite element analysis. The finite element formulation suggested in [28] is capable of analyzing panels under thermal loads. The deformations considered are so small (on the order of a few micrometers) that the correction u_i at any point can be assumed:

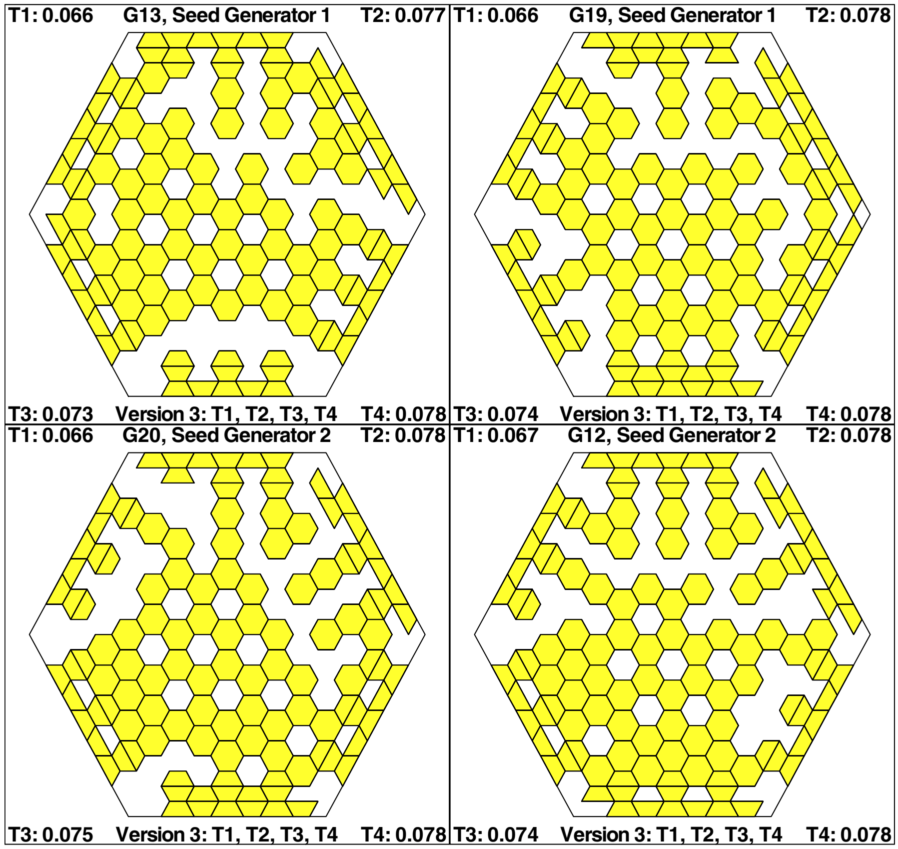


Fig. 6 RMS error and optimal location of 121 piezoelectric actuators.

Table 6 The rms errors corresponding to optimal placement (15 actuators)

Generations of inner loop	rms for initial evaluation		rms for 5000 evaluations		rms for 10,000 evaluations		rms for 15,000 evaluations	
	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2
8			0.42783	0.41276	0.41357	0.40926	0.40345	0.40905
9			0.40495	**0.39477**	0.40010	**0.38528**	0.39804	**0.38188**
10			**0.39957**	0.47201	0.39087	0.44066	0.38581	0.43244
11			0.40130	0.43442	**0.38671**	0.41774	0.38314	0.38577
12			0.41044	0.42707	0.39972	0.42327	**0.38018**	0.41599
13			0.41505	0.44012	0.40725	0.41871	0.40618	0.41636
14	1.66981	1.13175	0.40621	0.44519	0.39955	0.38707	0.39951	0.38342
15			0.41498	0.43830	0.41037	0.43830	0.40985	0.42657
16			0.40793	0.44148	0.40190	0.42044	0.40159	0.41474
17			0.41550	0.41523	0.40134	0.41455	0.39573	0.40850
18			0.41861	0.43983	0.40661	0.41402	0.40340	0.41295
19			0.41356	0.41859	0.41084	0.40415	0.40651	0.40415
20			0.41257	0.41963	0.40811	0.40114	0.39851	0.39657

$$u_i = \sum_{j=1}^{n} \alpha_{ij} V_j \tag{2}$$

where the control input V_j is applied across the jth piezoelectric strip and α_{ij} is defined as the deformation caused at node i due to a unit voltage applied across the jth piezoelectric strip alone.

A matrix of influence coefficients of size $m \times n$ is obtained from the finite element model by applying a unit voltage across each of the piezoelectric strips, one at a time. A measure of the overall deviation or the rms error is given by

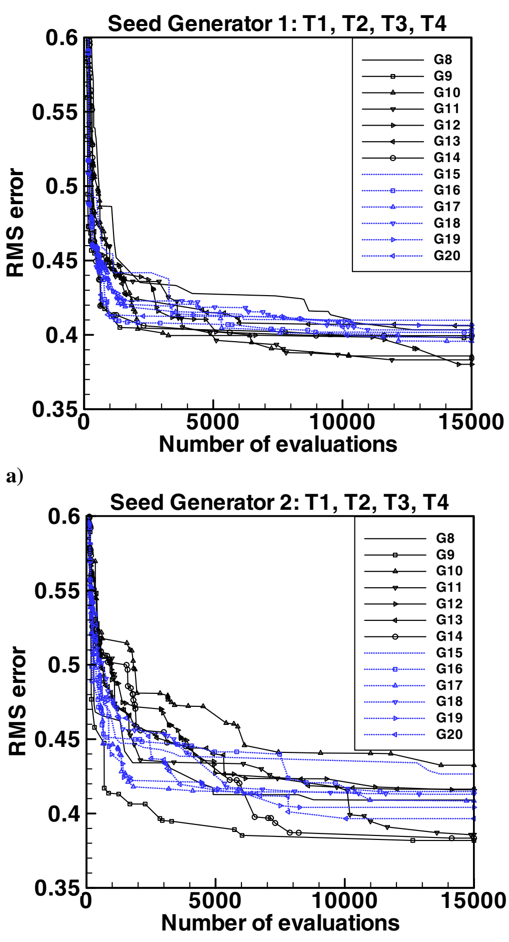


Fig. 7 Performance of the GAs: 15 actuators.

$$E = E(T, L, V) = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(w_i + u_i)^2}$$
$$= \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(w_i + \sum_{j=1}^{n}\alpha_{ij}V_j\right)^2} \tag{3}$$

To obtain the best correction, setting $\partial E/\partial V_k = 0$ gives

$$\sum_{i=1}^{m}\left(w_i + \sum_{j=1}^{n}\alpha_{ij}V_j\right)\alpha_{ik} = 0,$$

$$\text{i.e., } [A]\{V\} = \{b\} \quad \text{where } A_{kj} = \sum_{i=1}^{m}\alpha_{ij}\alpha_{ik}, \quad \boldsymbol{b}_k = -\sum_{i=1}^{m} w_i\alpha_{ik} \tag{4}$$

For each set of locations, we can get the optimal voltages to minimize rms error. Different settings have different optimal voltages and corresponding minimum rms error. For one type of thermal distortion we can find a set of locations and corresponding voltages that together minimize the minimum rms error, i.e., of the form

$$E = \min_L \min_V E(T, L, V) \tag{5}$$

For multiple types of thermal distortions, the optimization problem is to find a set of locations and corresponding voltages that minimizes the maximum of the minimum rms errors for all the different distortions, i.e., of the form

$$E = \min_L \max_T \min_V E(T, L, V) \tag{6}$$

VI. Results and Discussions

In this section, the performance of our latest micro-GA version 3 is studied through extensive numerical experiments. A high-performance Sun machine with 17 processors is used to do this research through a coarse-grain parallel computation to shorten the time of study. In the following Secs. VI.A–VI.E, we present the results corresponding to the test cases of choosing 30, 121, 15, 60, and 90 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$. The optimal voltages corresponding to optimal placements for these cases are available in [26]. For all cases, the GA parameters are chosen the same, as follows: initial population size is 10, population size is 5, scale is 0.5, random is 0, crossover rate is 0.5, mutation rate is 0.01, restart control parameter different level is 0.0, number of best mutation bits is 2, and number of evaluations is 15,000.

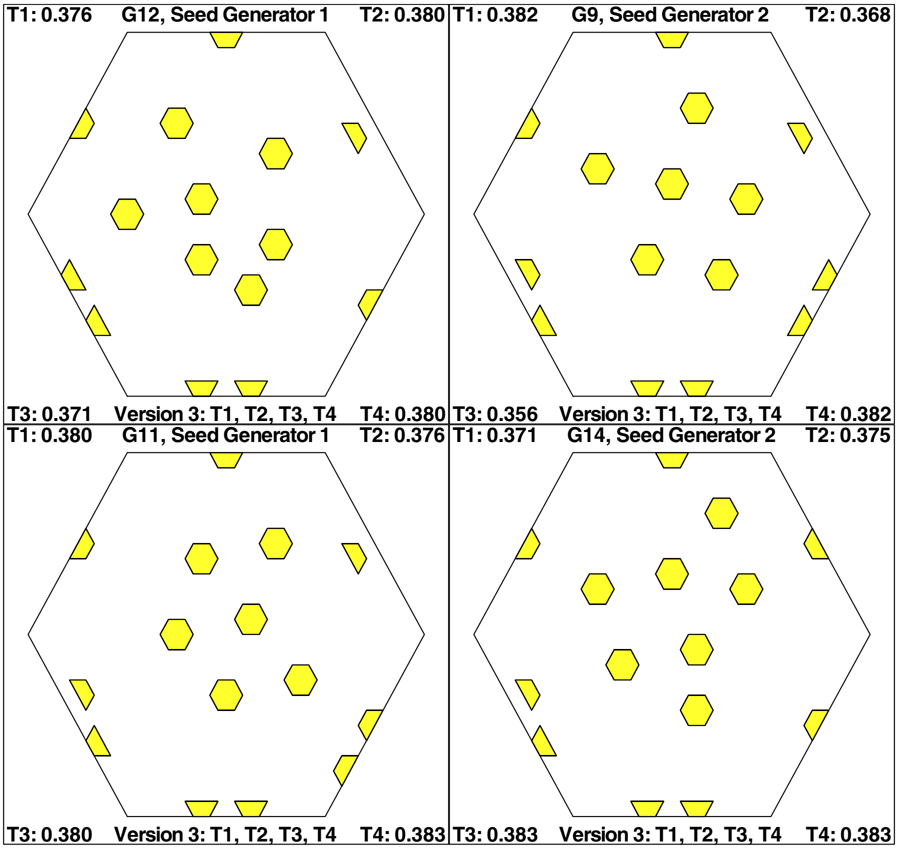


Fig. 8 RMS error and optimal location of 15 piezoelectric actuators.

At the end of this section, we briefly compare the performance of our GAs with WOBI and ESPS in the case of 15 actuators under a single type of thermal load $T1$ and multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$, respectively.

A. Case 1: $n = 30$ Actuators

The rms errors corresponding to the optimal placement of 30 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are shown in Table 4. The lowest rms value in each column is highlighted in the table. The convergence performance of the GAs in this case is presented in Fig. 3. The best four solutions to the optimal placement of 30 actuators are presented in Fig. 4. Two observations can be made from these results:

1) Seed generator 1 is better than seed generator 2 for all three numbers of evaluations used for termination.

2) The best number of generations for the inner loop depends on the number of evaluations used before termination. For example, if this number is 5000, the best number of generations used in the inner loop is 9, but if this number is 15,000, the best number of generations in the inner loop is 20.

B. Case 2: $n = 121$ Actuators

The rms errors corresponding to the optimal placement of 121 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are shown in Table 5. The lowest rms value in each column is highlighted in the table. The convergence performance of the GAs in this case is presented in Fig. 5. The best four solutions to the optimal

Table 7 The rms errors corresponding to optimal placement (60 actuators)

Generations of inner loop	rms for initial evaluation		rms for 5000 evaluations		rms for 10,000 evaluations		rms for 15,000 evaluations	
	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2
8			0.14527	0.14164	0.13996	0.13690	0.13850	0.13513
9			0.14180	0.14356	0.13861	0.14016	0.13565	0.13602
10			0.14266	0.14198	0.13815	0.13931	0.13558	0.13880
11			0.14130	0.13831	0.13858	0.13606	0.13564	0.13465
12			0.14425	0.13835	0.13986	0.13792	0.13769	0.13649
13			0.14059	0.15004	0.13802	0.13998	0.13697	0.13560
14	0.33310	0.34697	0.14780	**0.13319**	0.13848	**0.13319**	0.13797	0.13291
15			0.14183	0.14884	0.13707	0.14535	0.13540	0.14303
16			0.14568	0.14158	0.14355	0.14049	0.14311	0.14020
17			**0.14036**	0.14150	**0.13439**	0.13463	**0.13358**	**0.13164**
18			0.14834	0.13944	0.14438	0.13696	0.14244	0.13581
19			0.14763	0.13736	0.14030	0.13681	0.13810	0.13681
20			0.14293	0.14318	0.13889	0.13739	0.13441	0.13559

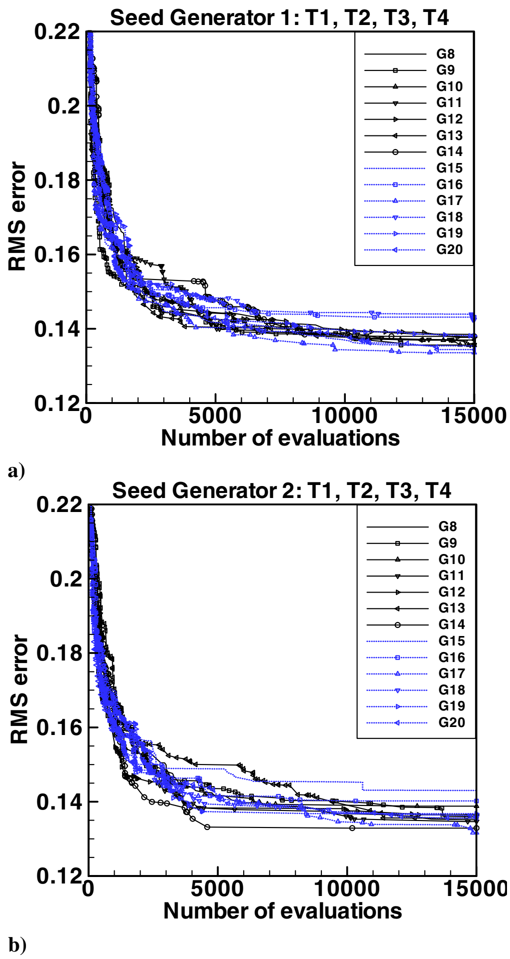


Fig. 9 Performance of the GAs: 60 actuators.

placement of 121 actuators are presented in Fig. 6. For this case, the following observations can be made:

1) Which seed generator is better depends on the number of evaluations used for termination. For example, if 5000 or 10,000 evaluations are chosen as the termination condition, seed generator 2 is better than seed generator 1, but if 15,000 evaluations are chosen as the termination condition, the conclusion is the reverse. However, the differences between the results obtained from two seeds are not substantially different.

2) The best number of generations for the inner loop depends on the number of evaluations used before termination. For example, if 5000 or 10,000 evaluations are performed before termination, the best number of generations used in the inner loop is 20, but if 15,000 evaluations are used as the termination condition, the number is 13.

The computation time in the case of 121 actuators is about one week per job running on one processor of the Sun machine. It should be noted here that the computation includes the actuator stiffness. The advantage of using the Sun machine is that we can submit multiple jobs running on different processors simultaneously, and so we can study the performance of GAs with different parameter settings at the same time and significantly reduce the time of investigation. Depending on the load of the computer and the number of jobs we can run simultaneously, the time of the investigation can be reduced by a factor of n. For example, if we can simultaneously submit 7 jobs, we maybe reduce the time of investigation by a factor of 7, at best.

C. Case 3: $n = 15$ Actuators

The rms errors corresponding to the optimal placement of 15 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are shown in Table 6. The lowest rms value in each column is highlighted in the table. The convergence performance of the GAs in this case is presented in Fig. 7. The best four solutions to the optimal placement of 15 actuators are presented in Fig. 8. For this case, the following observations can be made:

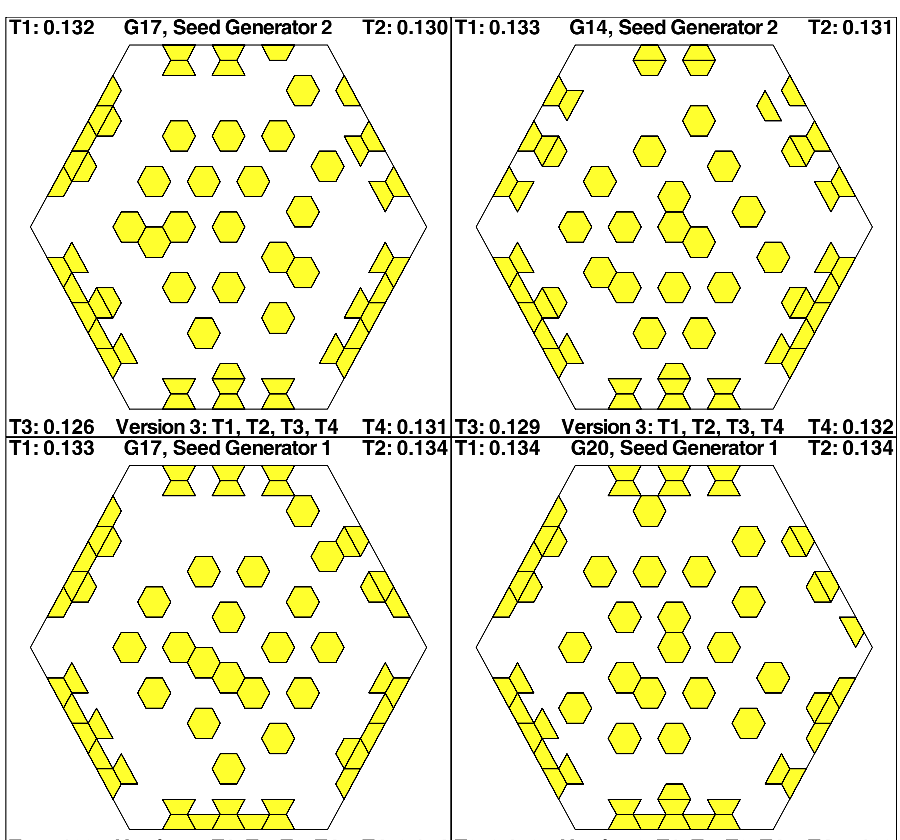


Fig. 10 RMS error and optimal location of 60 piezoelectric actuators.

Table 8 The rms errors corresponding to optimal placement (90 actuators)

Generations of inner loop	rms for initial evaluation		rms for 5000 evaluations		rms for 10,000 evaluations		rms for 15,000 evaluations	
	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2	Seed generator 1	Seed generator 2
8			0.10069	0.09227	0.09348	0.08907	0.09058	0.08658
9			0.09104	0.09300	0.08775	0.08818	0.08625	0.08724
10			0.09118	0.09243	**0.08657**	0.08772	**0.08564**	0.08632
11			0.09263	0.08981	0.08831	0.08839	0.08733	0.08705
12			0.09141	0.08856	0.09018	0.08713	0.08970	0.08654
13			0.09096	0.09069	0.08737	0.08661	0.08731	0.08658
14	0.23206	0.24537	0.09143	0.09052	0.08766	0.08896	0.08667	0.08748
15			0.09060	0.08969	0.08766	0.08793	0.08714	0.08687
16			0.09003	**0.08824**	0.08701	0.08652	0.08570	0.08612
17			0.08990	0.09110	0.08689	0.08858	0.08644	0.08701
18			**0.08924**	0.09280	0.08763	**0.08403**	0.08666	**0.08390**
19			0.08950	0.09213	0.08679	0.08757	0.08581	0.08702
20			0.08941	0.09083	0.08674	0.08666	0.08659	0.08539

1) Which seed generator is better depends on the number of evaluations used for termination. For example, if 5000 or 10,000 evaluations are chosen as the termination condition, seed generator 2 is better than seed generator 1, but if 15,000 evaluations are chosen as the termination condition, the conclusion is the reverse.

2) The best number of generations in the inner loop for each seed generator is different and may be dependent on the number of evaluations used for termination. For example, for seed generator 1, the best number of generations used in the inner loop is 10, 11, and 12 corresponding to the 5000, 10,000, and 15,000 evaluations as the termination condition, respectively; for seed generator 2, the best number of generations used in the inner loop is consistently 9 for all three termination conditions. The best number of generations in the inner loop is less than 14 for all three termination conditions. The range of the rms errors for seed generator 2 is obviously larger than that for seed generator 1.

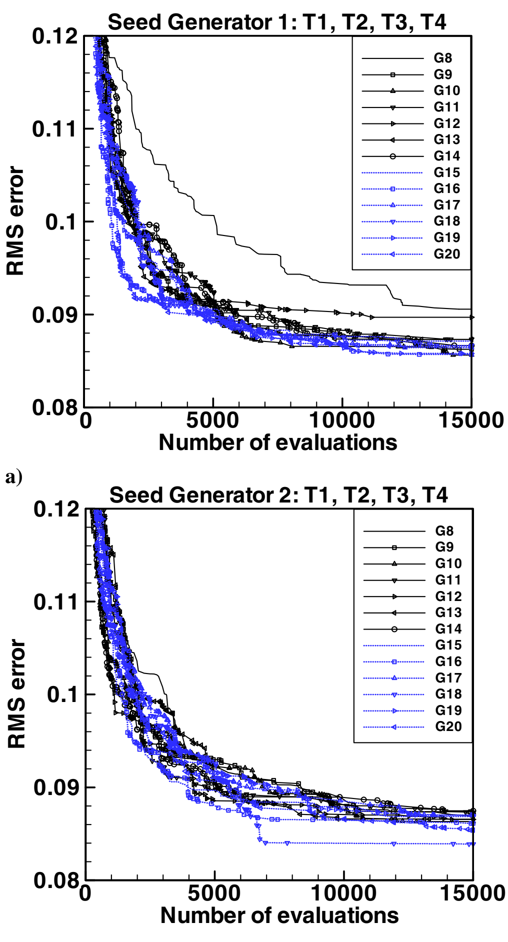


Fig. 11 Performance of the GAs: 90 actuators.

D. Case 4: $n = 60$ Actuators

The rms errors corresponding to the optimal placement of 60 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are shown in Table 7. The lowest rms value in each column is highlighted in the table. The convergence performance of the GAs in this case is presented in Fig. 9. The best four solutions to the optimal placement of 60 actuators are presented in Fig. 10. For this case, the following observations can be made:

1) The best number of generations in the inner loop for seed generator 1 is uniformly 17 for all three numbers of evaluations used for termination.

2) The best number of generations in the inner loop for seed generator 2 is 14 for the termination condition of 5000 and 10,000 evaluations and is 17 for the termination condition of 15,000 evaluations. For seed generator 2 and the termination condition of 15,000 evaluations, the optimal number 17 for the case of 60 actuators is different from those in the cases of 30 and 121 actuators (19 for the case of 30 actuators and 20 for the case of 121 actuators).

3) Which seed generator is better for a specific run is dependant on the number of evaluations. For example, the better seed generator is seed generator 1 for 5000 and 10,000 evaluations but seed generator 2 for 15,000 evaluations for the specific run of generations for the inner loop equal to 17.

4) All the best results for the three termination conditions of 5000, 10,000, and 15,000 evaluations are obtained by seed generator 2. This is quite different from the case of 30 actuators.

E. Case 5: $n = 90$ Actuators

The rms errors corresponding to the optimal placement of 90 actuators under multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are shown in Table 8. The lowest rms value in each column is highlighted in the table. The convergence performance of the GAs in this case is presented in Fig. 11. The best four solutions to the optimal placement of 90 actuators are presented in Fig. 12. For this case, the following observations can be made:

1) When the number of generations in the inner loop is 18, seed generator 2 in both cases of 10,000 and 15,000 evaluations shows much more superior performance than seed generator 1.

2) The best number of generations in the inner loop is dependent on not only the number of evaluations used for termination but also on which seed generator is used. For example, in the case of seed generator 1, the best number of generations in the inner loop is 18, 10, and 10 corresponding to the 5000, 10,000, and 15,000 evaluations as the termination condition, respectively. In the case of seed generator 2, the best number of generations in the inner loop is 16, 18,

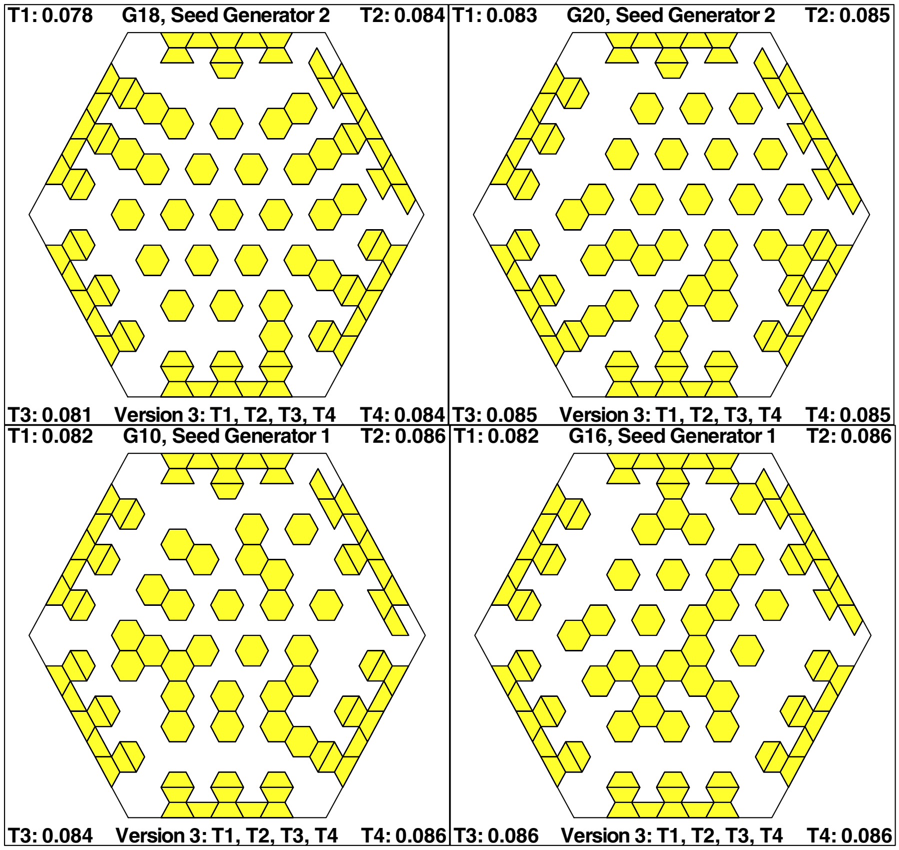


Fig. 12 RMS error and optimal location of 90 piezoelectric actuators.

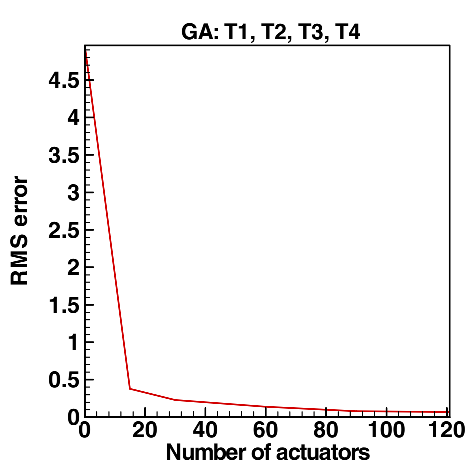


Fig. 13 RMS error vs number of actuators used in the optimal location.

and 18 corresponding to the 5000, 10,000, and 15,000 evaluations as the termination condition, respectively.

3) For the number of generations in the inner loop, the optimal number 18 in the case of seed generator 2 corresponding to the 10,000 and 15,000 evaluations is also different from those in the cases of 15, 30, 60, and 121 actuators.

4) The best solution for the case of 90 actuators is very close to that for the case of 121 actuators in terms of rms errors. Without correction, the rms value is 4.96 for thermal load $T1$, 3.07 for thermal load $T2$, 1.88 for thermal load $T3$, and 3.34 for thermal load $T4$. Therefore, the maximum rms value without correction is 4.96. With possible best correction, the rms value of the error for the minimum distortion is around 0.07 for 121 actuators, 0.08 for 90 actuators, 0.14 for 60 actuators, 0.23 for 30 actuators, and 0.38 for 15 actuators. The variation of rms error with the number of actuators used is approximately shown in Fig. 13. From this figure, we see that, although the rms error reduces with the number of actuators, the reduction of rms error may not be worth the effort of increasing the number of actuators.

In summary, the best result for each case of the different number of actuators is shown in Table 9.

Table 9 The rms error from all cases of actuators (best results only)

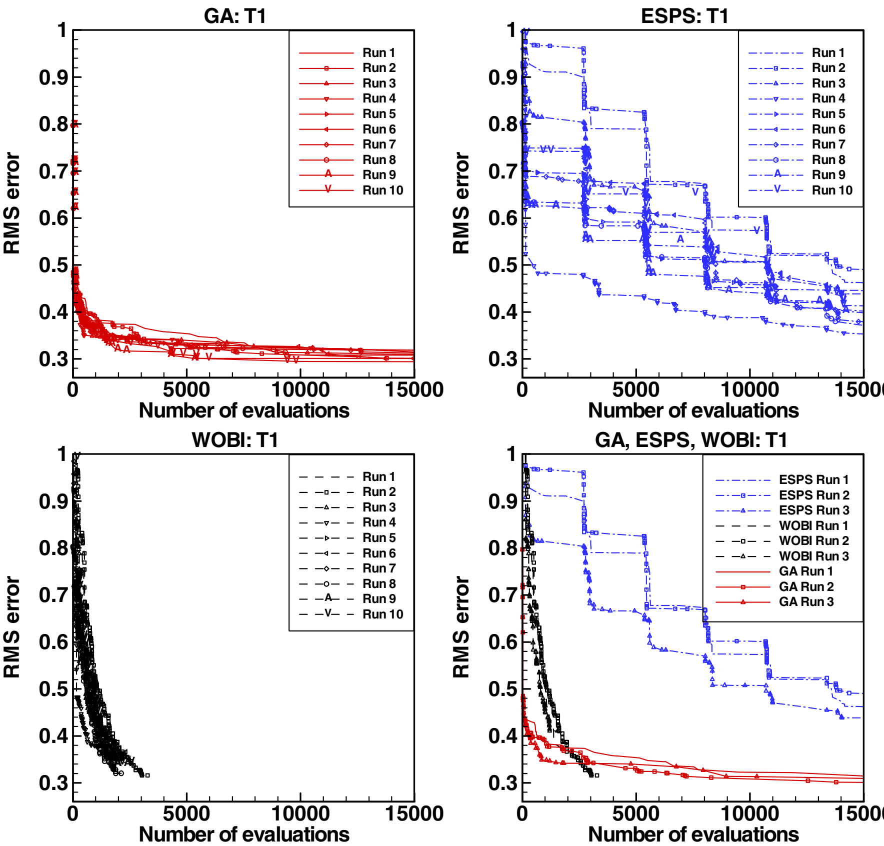
Number of actuators	Generations of inner loop	Seed generator no.	rms error for number of various thermal loads			
			$T1$	$T2$	$T3$	$T4$
15	12	1	0.37619	0.37962	0.37087	0.38018
30	20	1	0.22012	0.22819	0.22276	0.22882
60	17	2	0.13164	0.13028	0.12643	0.13118
90	18	2	0.07753	0.08390	0.08055	0.08382
121	13	1	0.06621	0.07732	0.07257	0.07788

Table 10 Comparison of WOBI, ESPS, and GAs in the case of 15 actuators (seed generator 1) on the thermal load $T1$ (neglecting strip stiffness)

Run no.	rms for initial evaluation	rms for 1000 evaluations	rms for 10,000 evaluations	rms for 15,000 evaluations	Range of rms for the first 40 RUNS
			WOBI		
1	1.670925	0.508452	0.366545	0.366545	0.410790–0.315454
2	2.008086	0.518876	0.315838	0.315838	
3	1.436401	0.456703	0.410790	0.410790	
4	0.797028	0.379624	0.361469	0.361469	
5	0.925741	0.434414	0.351240	0.351240	
6	1.080954	0.446338	0.366124	0.366124	
7	1.080309	0.437583	0.349820	0.349820	
8	0.803229	0.392083	0.320552	0.320552	
9	0.721101	0.413513	0.340376	0.340376	
10	1.448242	0.470019	0.343649	0.343649	
			ESPS		
1	1.670925	0.913176	0.574384	0.462974	0.334335–0.292368
2	2.008086	0.967427	0.601831	0.491210	
3	1.436401	0.814665	0.507459	0.438397	
4	0.797028	0.482485	0.388202	0.355070	
5	0.925741	0.696317	0.462594	0.406930	
6	1.080954	0.749297	0.528433	0.446338	
7	1.080309	0.688507	0.459364	0.379454	
8	0.803229	0.633728	0.451879	0.379902	
9	0.721101	0.625868	0.444337	0.401664	
10	1.448242	0.742147	0.506790	0.413956	
			GAs		
1	1.670925	0.390345	0.322724	0.314750	0.299658–0.287657
2	1.670925	0.391254	0.311184	0.301496	
3	1.670925	0.348705	0.313745	0.313425	
4	1.670925	0.350171	0.330336	0.319885	
5	1.670925	0.373046	0.322582	0.319642	
6	1.670925	0.359572	0.326310	0.300929	
7	1.670925	0.382446	0.324182	0.315101	
8	1.670925	0.357506	0.308492	0.308492	
9	1.670925	0.355651	0.301156	0.301156	
10	1.670925	0.361758	0.294486	0.294486	

Table 11 Comparison of WOBI, ESPS, and GAs in the case of 15 actuators (seed generator 1) on thermal loads $T1$, $T2$, $T3$, and $T4$ (neglecting strip stiffness)

Run no.	rms for initial evaluation	rms for 1000 evaluations	rms for 10,000 evaluations	rms for 15,000 evaluations	Range of rms for the first 40 runs
			WOBI		
1	1.670925	0.709555	0.454307	0.454307	0.566157–0.413216
2	2.008086	0.631938	0.418519	0.418519	
3	1.436401	0.641500	0.488944	0.488944	
4	0.797028	0.427736	0.427736	0.427736	
5	0.925741	0.524296	0.439216	0.439216	
6	1.080954	0.606985	0.483070	0.483070	
7	1.080309	0.546180	0.441704	0.441704	
8	0.968807	0.494342	0.421853	0.421853	
9	0.840608	0.566460	0.566157	0.566157	
10	1.448242	0.564475	0.423584	0.423584	
			ESPS		
1	1.670925	1.036378	0.874500	0.633940	0.423585–0.370179
2	2.008086	0.967427	0.644622	0.573124	
3	1.436401	0.884101	0.642493	0.582354	
4	0.797028	0.517246	0.433307	0.419940	
5	0.925741	0.745481	0.584407	0.513788	
6	1.080954	0.826264	0.588146	0.545529	
7	1.080309	0.820592	0.578790	0.509677	
8	0.968807	0.833939	0.547014	0.487450	
9	0.840608	0.707136	0.581718	0.525421	
10	1.448242	0.742147	0.594218	0.549260	
			GAs		
1	1.670925	0.459790	0.396555	0.393750	0.393148–0.361743
2	1.670925	0.427220	0.399434	0.393077	
3	1.670925	0.448521	0.380142	0.380142	
4	1.670925	0.419861	0.394468	0.386985	
5	1.670925	0.455181	0.399857	0.396110	
6	1.670925	0.421537	0.397604	0.393516	
7	1.670925	0.438067	0.402400	0.398655	
8	1.670925	0.431978	0.393097	0.392616	
9	1.670925	0.434977	0.410906	0.409302	
10	1.670925	0.413924	0.412856	0.387242	



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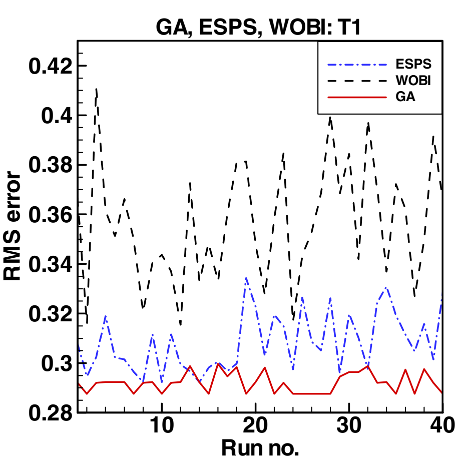

b)

Fig. 14 Performance of GA, ESPS, and WOBI: 15 actuators and seed generator 1.

F. Comparison of Genetic Algorithms with Worst-Out-Best-In and Exhaustive Single Point Substitution

The comparison of GAs with WOBI and ESPS is presented here in the case of 15 actuators under a single type of thermal load $T1$ and multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$, respectively. For each method we perform 40 runs. Seed generator 1 is used in all runs. The 40 seeds for WOBI and ESPS are the same as those in the initial population of GAs. They are the first 40 individuals randomly generated by seed generator 1. The 40 seeds are divided into 4 groups of 10 individuals: seeds 1 to 10, seeds 11 to 20, seeds 21 to 30, and seeds 31 to 40. Each group is used as an initial population of 10 runs of the GA, made by setting the number of generations in the inner loop from 11 to 20. To speed up the computation, we neglect the stiffness of the actuators but include the actuator stiffness in the final presentation of best layouts. We first run these programs in the case of thermal load $T1$ and then in the case of thermal loads $T1$, $T2$, $T3$, and $T4$. The number of evaluations for the convergence of WOBI and ESPS in the case of 15 actuators is within 10^4 and 10^5 for these 40 runs, respectively. The number of evaluations for GA termination is set as 10^7, though some runs may not need so many evaluations to reach the best solution. For example, it takes 130,458 evaluations for GA run 10 to reach the best solution (rms $= 0.287$) in the case of thermal load $T1$. The rms error for 15,000 evaluations in the first 10 runs of each method and the range of rms errors for the best solutions

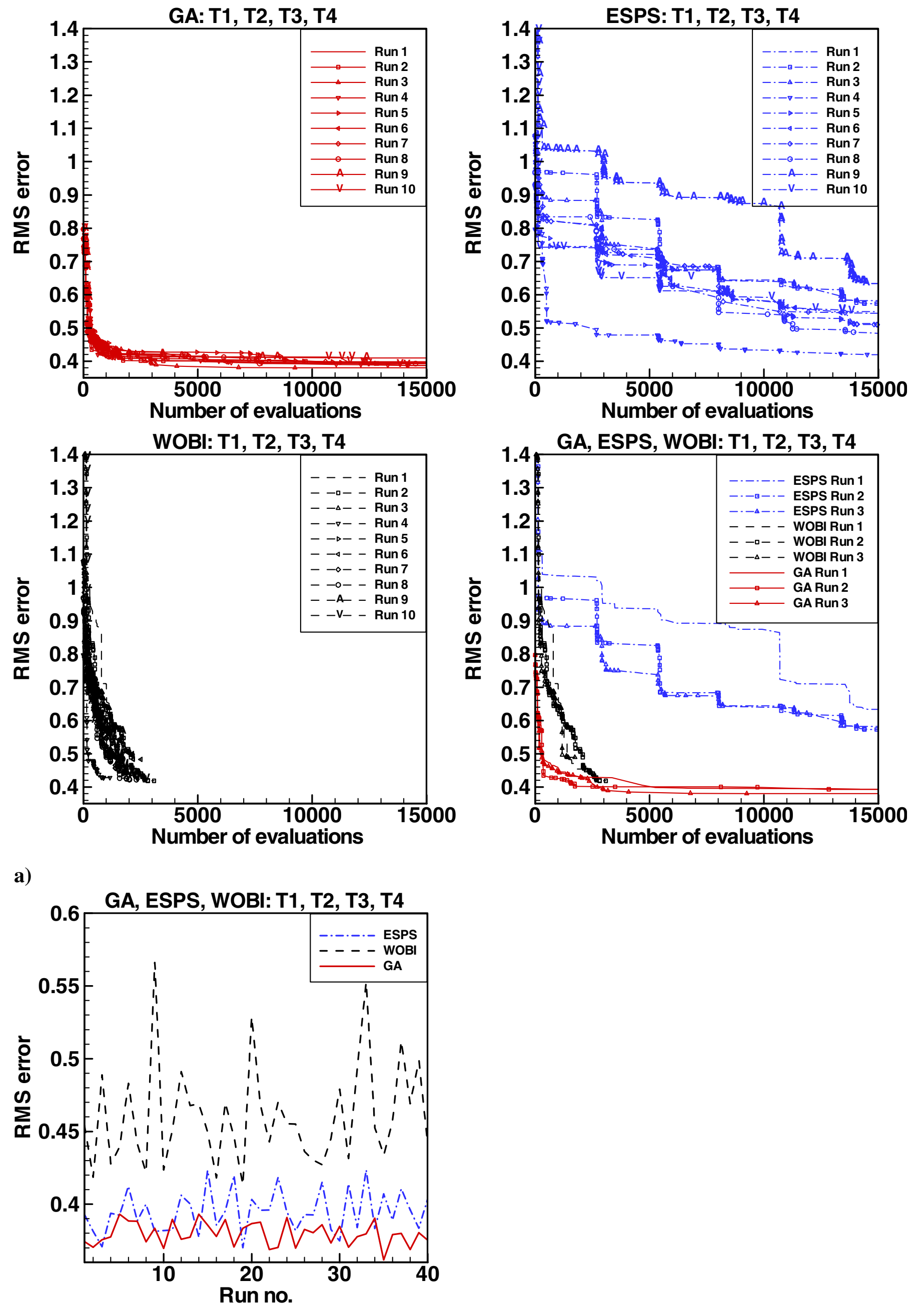

Fig. 15 Performance of GA, ESPS, and WOBI: 15 actuators and seed generator 1.

in 40 runs of each method under a single type of thermal load $T1$ and multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ are listed in Tables 10 and 11, respectively. The performance of these algorithms is shown in Figs. 14 and 15, respectively. The best layout for 15 actuators found by the optimization under a single type of thermal load $T1$ and multiple types of thermal loads $T1$, $T2$, $T3$, and $T4$ is shown in Figs. 16 and 17, respectively. From these results, we can see that for the group of runs GAs usually converge faster in the first few thousand evaluations and are more likely to find the better solutions than WOBI and ESPS as long as GAs are allowed to run enough time and that WOBI and ESPS usually get trapped in a local optimum. More comparison of GAs with WOBI and ESPS in the case of 30 and 121 actuators is ongoing and will be reported in another paper [29].

VII. Conclusions

Through extensive numerical experiments, we found that the most distinct nature of genetic algorithms is randomness and robustness. Here we are not considering theoretical genetic algorithms with an unlimited unrealistic very large number of evaluations. Rather, we are considering practical genetic algorithms with a limited number of evaluations (15,000 evaluations). By randomness, we mean that we cannot precisely predict the performance of genetic algorithms in future evaluations based on their performance in previous evaluations. By robustness, we mean that, although genetic algorithms have their random nature, the range of their results in terms of fitness is usually small from different runs after a certain number of evaluations and that they are capable of finding a very good solution through a group of runs by simply adjusting the parameter setting or using another random seed generator. It should be noted here that, in general, the robustness of GAs might be problem dependent. Whether GAs can be successfully applied to some quite different problems depends on several factors such as the nature of the problem, how a GA is designed, the choice of parameters, how much computational resource is available, etc. Among them, how a GA is designed is most important. Our research

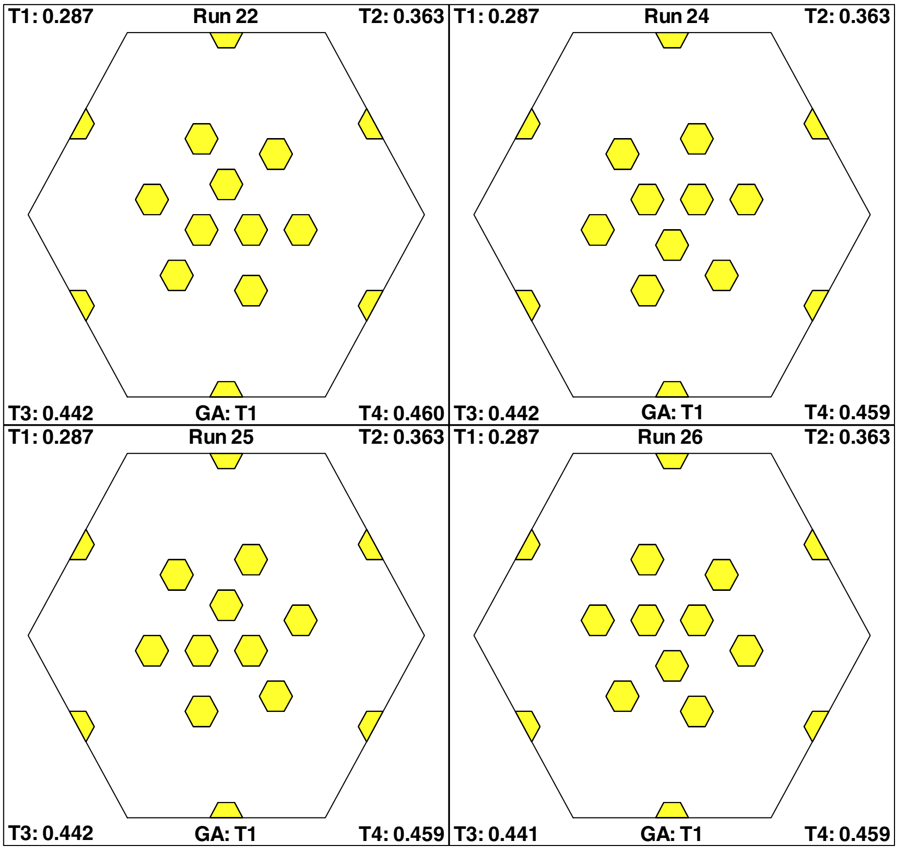

Fig. 16 RMS error and optimal location of 15 piezoelectric actuators: seed generator 1.

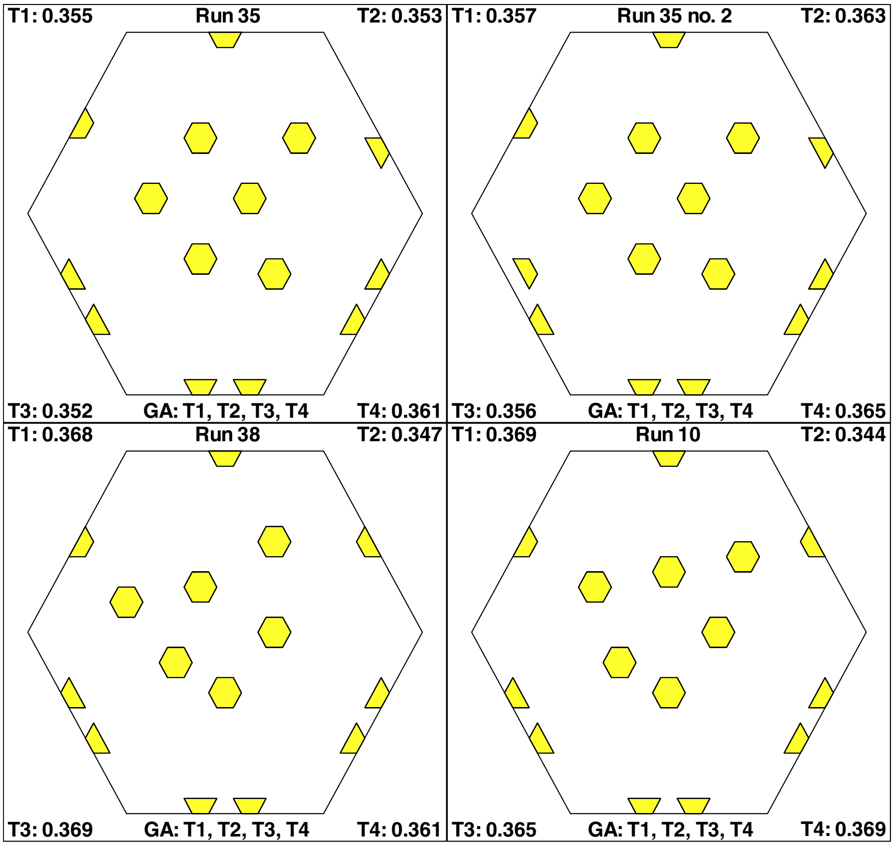

Fig. 17 RMS error and optimal location of 15 piezoelectric actuators: seed generator 1.

also shows that the best parameter setting of GAs is dependent not only on the number of evaluations used for termination but also on which seed generator is used. In addition, the best parameter setting of GAs is different as the number of actuators changes. Because of their nonlinear convergence, the genetic algorithms can be used to get an appropriate solution with a few hundred evaluations. The genetic algorithms are also easy to use to generate an alternative solution. To get high-quality solutions in the design of complex adaptive structures using finite element analysis and genetic algorithm optimization, the multiple runs including different random seed generators are necessary. The time of the investigation can be significantly reduced using a very coarse-grain parallel computing, by simply running multiple jobs at the same time. Our research also shows that, although the rms error reduces with the number of actuators, the reduction of rms error may not be worth the effort of increasing the number of actuators. From the results of the optimal voltages we can see that some voltages are extremely high. This means that further investigations are needed to consider the optimal problems including voltage constraints. From the limited comparison of GAs, WOBI, and ESPS, we can see that for the group of runs GAs usually converge faster in the first few thousand evaluations and are more likely to find a better solution than WOBI and ESPS as long as GAs are allowed to run enough time. Overall, the methodology of using finite element analysis and genetic algorithm optimization is a very good approach, and GA version 3 is an efficient, reliable, and robust optimization tool for the challenging problem, optimal placements of a large number of actuators in the design of the next generation of adaptive structures.

Acknowledgments

We would like to thank Raphael T. Haftka at the University of Florida and the reviewers of this journal for their insightful and helpful comments during the review of the paper.

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